Stuff

Small Geometric Puzzle **August 7, 2021 (newest version)**

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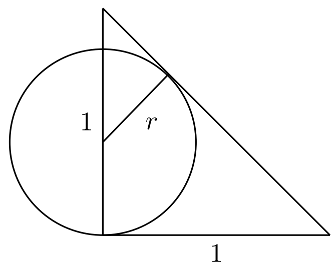
Abstract

A small geometric puzzle, solved analytically.

Problem Statement Consider the following geometric problem in $\mathbb{R}^2$:



Where the circle is tangent to the line containing the hypothenuse of the triangle. What is the value of r? It is clear that the circle is centered at $(0, r)$ as it intersects with $(0, 0)$. I will give an analytic solution to this problem. r is unknown, but let it be fixed. We model the above halfcircle by constructing a function f from the circle equation in $[0, r]$:

$$\sqrt{x^2 + (y - r)^2} = r \implies f(x) = \sqrt{r^2 - x^2} + r$$

The hypothenuse can be modelled by the equation $-x + 1$, thus we want to obtain an r such that there is a single intersection of that curve in this interval:

$$\sqrt{r^2 - x^2} + r = -x + 1 \implies 0 = x^2 + (r - 1)x + \left(-r + \frac{1}{2}\right)$$

We solve after x:

$$x_{1,2} = \frac{1 - r}{2} \pm \sqrt{\frac{(r - 1)^2}{4} + r - \frac{1}{2}}$$

For obtaining a single solution, the latter term must be zero. Thus continuing with r:

$$\frac{(r - 1)^2}{4} + r - \frac{1}{2} = 0 \implies r^2 + 2r - 1 = 0$$

Solving for r obtains two solutions $-1 \pm \sqrt{2}$, of which only the positive one is of interest. Thus, the solution is $r = -1 + \sqrt{2}$.